

What does a non-vanishing neutrino mass have to say about the strong CP problem?

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A new solution to the strong CP problem with distinct experimental signatures at the LHC is proposed. It is based on the Yukawa interactions between mirror quarks, Standard Model (SM) quarks and Higgs singlets. (Mirror quarks and leptons which include non-sterile right-handed neutrinos whose Majorana masses are proportional to the electroweak scale, form the basis of the EW- ν_R model.) The aforementioned Yukawa couplings can in general be complex and can contribute to $\text{Arg Det}M$ ($\bar{\theta} = \theta_{QCD} + \text{Arg Det}M$) at tree-level. The crux of matter in this manuscript is the fact that *no matter how large* the CP-violating phases in the Yukawa couplings might be, $\text{Arg Det}M$ can remain small i.e. $\bar{\theta} < 10^{-10}$ for reasonable values of the Yukawa couplings and, in fact, vanishes when the VEV of the Higgs singlet (responsible for the Dirac part of the neutrino mass in the seesaw mechanism) vanishes. The smallness of the contribution to $\bar{\theta}$ is *principally due* to the smallness of the ratio of the two mass scales in the seesaw mechanism: the Dirac and Majorana mass scales.

It is a well-known fact that, although CPT appears to be respected as a symmetry of nature, CP and T are not as weak interaction experiments have shown us. Furthermore, studies of the QCD vacuum, the so-called θ -vacuum, revealed that an additional CP-violating term is added to the Lagrangian in the form $\theta_{QCD} (g_3^2/32\pi^2) G_a^{\mu\nu} \tilde{G}_{a\mu\nu}^{\tilde{}}$. In addition, the electroweak sector contributes another similar term through quark mass matrices so that the total θ is now $\bar{\theta} = \theta_{QCD} + \text{Arg Det}M$.

Constraints coming from the absence of the neutron electric dipole moment give $\bar{\theta} < 10^{-10}$ [1]. This is the famous strong CP problem: why $\bar{\theta}$ which is the sum of the contributions from the strong and weak sectors should be so small. Several lines of approach toward a solution to the strong CP problem have been proposed. The most famous one is the Peccei-Quinn axion [2] where a new global $U(1)_{PQ}$ was added and where $\bar{\theta}$ is driven dynamically to zero. The axion is still elusive and its search is going on. A very early class of models solving the strong CP problem without the axion and using either soft CP breaking or simply P and T invariance can be found in [3]. Another line of approach [4] is to assume CP conservation of the Lagrangian so that $\bar{\theta} = 0$ at tree level and to postulate the existence of heavy fermions (within a Grand Unification context such as SU(5) or an extended gauge group $SM \times G$) to generate a non-vanishing $\bar{\theta}$ at loop levels. In this class of models, CP is spontaneously broken giving rise to potential problems with issues such

as domain walls.

Our approach to the strong CP problem is a non-axionic one and is more similar in spirit to the approach which assumed the presence of non-SM fermions, except for a few crucial differences. It *does not impose* CP conservation of the Lagrangian. Three questions that need to be addressed are the following: 1) If CP conservation is not imposed on the Lagrangian, what symmetry allows us to set the QCD θ_{QCD} to be equal to zero at tree level?; 2) Since CP can explicitly be violated by the complex Yukawa couplings, what prevents $\text{Arg Det}M$ from exceeding the upper bound of 10^{-10} ?; 3) Last but not least, can the solution be found *solely* within the gauge structure of the SM, namely $SU(3) \times SU(2) \times U(1)$?

The answers to the aforementioned three questions can be found in the EW- ν_R model [5] and will be elaborated below. All the ingredients for a solution to the strong CP problem are already contained in this model. However, a few key points about the EW- ν_R model need to be mentioned: 1) it avoids the Nielsen-Ninomiya no-go theorem [6] (which says that one cannot put the SM on the lattice without having mirror fermions interacting with the same W and Z bosons) by postulating the very existence of these mirror fermions. Note that mirror fermions are also motivated within the framework of E_6 with 27_L and 27_L^c representations [5]; 2) the right-handed neutrinos, being part of a right-handed mirror lepton doublet, are now *non-sterile* (or *fertile*) and obtain Majorana masses which are proportional to the electroweak scale and can be produced at the LHC and searched for by looking for like-sign dileptons events; 3) it satisfies the electroweak precision constraints as well as being able to accommo-

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date the 125-GeV scalar in an interesting way.

We start out with the one-generation case in the EW- ν_R model. This helps to separate the two issues, that of the strong CP violation and that of the weak CP violation present in the CKM matrix for three (or more) generations of quarks.

The gauge group is $SU(3)_C \times SU(2)_W \times U(1)_Y$. We have one generation of SM quarks: $q_L = (u_L, d_L)$, u_R, d_R , and one generation of mirror quarks: $q_R^M = (u_R^M, d_R^M)$, u_L^M, d_L^M . (The full model can be found in the EW- ν_R model [5].) For the purpose of this manuscript, we will focus on the Yukawa couplings:

$$\mathcal{L}_{mass} = g_u \bar{q}_L \tilde{\Phi}_2 u_R + g_d \bar{q}_L \Phi_2 d_R + g_{u^M} \bar{q}_R^M \tilde{\Phi}_{2M} u_L^M + g_{d^M} \bar{q}_R^M \Phi_{2M} d_L^M + H.c., \quad (1)$$

where $\tilde{\Phi}_{2,2M} \equiv \tau_2 \Phi_{2,2M}$ with $\Phi_{2,2M}$ being the two Higgs doublets of the extended EW- ν_R model [7].

$$\mathcal{L}_{mixing} = g_{Sq} \bar{q}_L \phi_S q_R^M + g_{Su} \bar{u}_L^M \phi_S u_R + g_{Sd} \bar{d}_L^M \phi_S d_R + H.c., \quad (2)$$

where ϕ_S is a Higgs singlet. The rationale for introducing the aforementioned degrees of freedom can be found in [5].

Notice that $g_u, g_d, g_{u^M}, g_{d^M}, g_{Sq}, g_{Su}$ and g_{Sd} can, in general be complex. If we absorb the phases into u_R, u_L^M, d_R and d_L^M to make the *diagonal* elements of the (2×2) up and down mass matrices *real* then the *off-diagonal* elements stay *complex*. Furthermore, a global symmetry was invoked in [5, 7] to ensure that the Yukawa couplings take the form as shown in Eq. (1). $\langle \Phi_2 \rangle = v_2$ and $\langle \Phi_{2M} \rangle = v_{2M}$ give non-vanishing masses to the SM and mirror quarks, namely m_u, m_d, M_u and M_d respectively. (From [5, 7], $v_2^2 + v_{2M}^2 + 8v_M^2 = (246 \text{ GeV})^2$ where v_M is the VEV of the Higgs triplet which gives the Majorana mass to ν_R .) The mass mixing between SM and mirror quarks comes from Eq. (2). Writing $g_{Sq} = |g_{Sq}| \exp(i\theta_q)$, $g_{Su} = |g_{Su}| \exp(i\theta_u)$ and $g_{Sd} = |g_{Sd}| \exp(i\theta_d)$ and with $\langle \phi_S \rangle = v_S$, we obtain the following mass matrices (m_u, M_u, m_d and M_d are real)

$$\mathcal{M}_u = \begin{pmatrix} m_u & |g_{Sq}| v_S \exp(i\theta_q) \\ |g_{Su}| v_S \exp(i\theta_u) & M_u \end{pmatrix}, \quad (3)$$

$$\mathcal{M}_d = \begin{pmatrix} m_d & |g_{Sq}| v_S \exp(i\theta_q) \\ |g_{Sd}| v_S \exp(i\theta_d) & M_d \end{pmatrix}. \quad (4)$$

One can now compute $\bar{\theta}$, namely

$$\bar{\theta} = \theta_{QCD} + \text{ArgDet}(\mathcal{M}_u \mathcal{M}_d). \quad (5)$$

Two important questions are in order.

- Can θ_{QCD} be *zero* at tree level?

To answer this question, we make use of the distinct feature of the EW- ν_R model which is *parity invariance* above the electroweak scale coming from the fact that one has both left- and right-handed fermions transforming in the same way under $SU(2)_W$ (hence the subscript "W" instead of "L"). Since $G_a^{\mu\nu} \tilde{G}_{\mu\nu}$ is *odd* under Parity, it follows that the QCD $\theta_{QCD} = 0$ at tree level. (A similar argument was used in the Left-Right symmetric model [3].)

- Since CP is explicitly violated in $\mathcal{M}_{u,d}$, could their *tree-level* contribution to $\text{ArgDet}(\mathcal{M}_u \mathcal{M}_d)$ be *naturally* small without fine-tuning the CP phases?

From Eq. (3,4), we obtain ($C_u \equiv m_u M_u$, $C_d \equiv m_d M_d$, $C_{Su} \equiv |g_{Sq}| |g_{Su}| v_S^2$ and $C_{Sd} \equiv |g_{Sq}| |g_{Sd}| v_S^2$)

$$\text{ArgDet}(\mathcal{M}_u \mathcal{M}_d) = \text{Arg}\{(C_u - C_{Su} \exp[i(\theta_q + \theta_u)])(C_d - C_{Sd} \exp[i(\theta_q + \theta_d)])\}. \quad (6)$$

Neglecting the term proportional to $C_{Su} C_{Sd}$ since (as we shall explain below) $C_{Su} C_{Sd} \ll C_u C_d, C_{Su} C_d, C_{Sd} C_u$, we obtain with $\theta_{Weak} \equiv \text{ArgDet}(\mathcal{M}_u \mathcal{M}_d)$

$$\theta_{Weak} \approx \tan^{-1} \frac{-(C_{Su} C_d \sin(\theta_q + \theta_u) + C_{Sd} C_u \sin(\theta_q + \theta_u))}{C_d C_u - C_{Su} C_d \cos(\theta_q + \theta_u) - C_{Sd} C_u \cos(\theta_q + \theta_u)} \quad (7)$$

Defining

$$r_u = \frac{C_{Su}}{C_u} = \frac{|g_{Sq}| |g_{Su}| v_S^2}{m_u M_u}, \quad (8)$$

$$r_d = \frac{C_{Sd}}{C_d} = \frac{|g_{Sq}| |g_{Sd}| v_S^2}{m_d M_d}, \quad (9)$$

Eq. (7) can now be put in a neater form

$$\theta_{Weak} \approx \frac{-(r_u \sin(\theta_q + \theta_u) + r_d \sin(\theta_q + \theta_d))}{1 - r_u \cos(\theta_q + \theta_u) - r_d \sin(\theta_q + \theta_d)} \quad (10)$$

- First, we notice from Eq. (10) that $\theta_{Weak} = 0$ when the VEV of the singlet Higgs vanishes i.e. when $v_S = 0$. This is valid for *any* value of the phases $\theta_{q,u,d}$.
- θ_{Weak} can also vanish if all the phase angles vanish or if $\theta_q = -\theta_u = -\theta_d$. Since these are special cases, we will not consider them here but will instead keep them arbitrary.

- As shown in [5], a non-vanishing value for v_S implies a non-vanishing Dirac mass of the neutrino participating in the seesaw mechanism i.e. $m_\nu = m_D^2/M_R$. From [5], $m_D = g_{Sl}v_S$ coming from the interaction $g_{Sl}\bar{l}_L\phi_S l_R^M + H.c.$ where $l_L = (\nu_L, e_L)$ and $l_R^M = (\nu_R, e_R^M)$. Here M_R is the Majorana mass of the right-handed neutrino coming from $g_M(l_R^{M,T}\sigma_2)(\nu\tau_2)\tilde{\chi}l_R^M$ where χ is a triplet Higgs with $Y/2 = 1$ and whose VEV is v_M .
- Since $M_R > M_Z/2 \sim 45$ GeV (from the Z-width constraint), one gets $m_D < 100$ keV [5].
- One can rewrite r_u and r_d as

$$r_u = \left(\frac{|g_{Sq}||g_{Su}|}{g_{Sl}^2}\right)\left(\frac{m_D^2}{m_u M_u}\right), \quad (11)$$

$$r_d = \left(\frac{|g_{Sq}||g_{Sd}|}{g_{Sl}^2}\right)\left(\frac{m_D^2}{m_d M_d}\right). \quad (12)$$

$r_{u,d} \ll 1$ if one assumes that $|g_{Sq}||g_{Su}| \leq g_{Sl}^2$,

- It is interesting to notice that, since the Majorana mass of the right-handed neutrinos is also proportional to the electroweak scale 246 GeV, r_u and r_d which will determine the size of θ_{Weak} have the following proportionality $r_u \propto m_\nu/m_u$; $r_d \propto m_\nu/m_d$ and vanish as $m_\nu \rightarrow 0$.
- One can now rewrite θ_{Weak} as

$$\theta_{Weak} \approx -(r_u \sin(\theta_q + \theta_u) + r_d \sin(\theta_q + \theta_d)). \quad (13)$$

- As discussed in [7], one expects the mirror quarks to be heavy. For the sake of estimation, we shall take $M_u \sim M_d \sim 400$ GeV. Furthermore, since we are dealing with the one-generation case, let us take the most extreme case, namely $m_u \sim 2.3$ MeV and $m_d \sim 4$ MeV. With the constraint $m_D < 100$ keV, one obtains the following bound

$$\begin{aligned} \theta_{Weak} < -10^{-8} \left\{ \left(\frac{|g_{Sq}||g_{Su}|}{g_{Sl}^2} \right) \sin(\theta_q + \theta_u) \right. \\ \left. + \left(\frac{|g_{Sq}||g_{Sd}|}{g_{Sl}^2} \right) \sin(\theta_q + \theta_d) \right\} \end{aligned} \quad (14)$$

- What does the inequality (14) imply? $|\theta_{Weak}| < 10^{-10}$ regardless of the values of the CP phases. Even if one had *maximal* CP violation in the sense that $\theta_q + \theta_u \sim \theta_q + \theta_d \sim \pi/2$, $|\theta_{Weak}| < 10^{-10}$ provided $|g_{Sq}| \sim |g_{Su}| \sim |g_{Sd}| \sim 0.1g_{Sl}$.

- This has interesting phenomenological implications concerning the searches for mirror quarks and leptons at the LHC [8]. In fact, constraints coming from $\mu \rightarrow e\gamma$ [9] and from μ - e conversion [10] indicate that $g_{Sl} < 10^{-4}$ which would imply in the present context that $|g_{Sq}| \sim |g_{Su}| \sim |g_{Sd}| < 10^{-5}$. This implies the possibility of observing the decays of mirror quarks and leptons from the process $f^M \rightarrow f + \phi_S$ (where f^M and f stand for mirror and SM fermions respectively) at *displaced vertices* (large decay lengths) because of the small Yukawa couplings.
- As already pointed out in [5], the mass mixing between SM and mirror quarks is tiny, being proportional to the ratio of neutrino to quark mass. For most practical purpose, the mass eigenstates are approximately pure SM and mirror states.

A full analysis will involve three generations and will be more complicated. As opposed to the one-generation case where we have a 2×2 matrix, we will now have a 6×6 matrix of the form

$$\mathcal{M}_u = \begin{pmatrix} M_u & M_{q_L q_R^M} \\ M_{u_R u_L^M} & M_{u^M} \end{pmatrix}, \quad (15)$$

$$\mathcal{M}_d = \begin{pmatrix} M_d & M_{q_L q_R^M} \\ M_{d_R d_L^M} & M_{d^M} \end{pmatrix}, \quad (16)$$

where each element of the above matrices are 3×3 matrices. The matrices $M_{q_L q_R^M}$, $M_{u_R u_L^M}$ and $M_{d_R d_L^M}$ contain matrix elements which are proportional to the VEV of the singlet Higgs field, namely v_S . As we have shown above for the one-generation case, these are much smaller than matrix elements of M_u , M_d , M_{u^M} and M_{d^M} . For this reason, those mass matrices can be diagonalized separately, neglecting mixing. Furthermore, we believe that the result for θ_{Weak} will not be too different for that given in Eq. (14).

We carried out an analysis based on a simplified version of the full model. (The full analysis is beyond the scope of the paper and will be presented elsewhere.) We assume that M_{u^M} and M_{d^M} are diagonal. The problem is now reduced to a diagonalization of a 4×4 matrix of the form

$$\tilde{\mathcal{M}}_{u,k} = \begin{pmatrix} M_u & M_{q_L q_R^M}^{i4} \\ M_{u_R u_L^M}^{4j} & m_{u^M,k} \end{pmatrix}, \quad (17)$$

where $i, j, k = 1, 2, 3$ and where $m_{u^M, k}$ denotes the mass of the k th up mirror quark. Similarly, one has

$$\tilde{\mathcal{M}}_{d, k} = \begin{pmatrix} M_d & M_{q_L q_R^M}^{i4} \\ M_{d_R d_L^M}^{4j} & m_{d^M, k} \end{pmatrix}. \quad (18)$$

For simplicity, let us assume $m_{u^M, k} = m_{u^M}$ and $m_{d^M, k} = m_{d^M}$. In a recent work, Ref. [11] constructs phenomenologically the up and down-quark mass matrices M_u and M_d which can reproduce the known phenomenology of the CKM matrix and quark masses. These matrices turn out to be Hermitian and have real determinants. A simple calculation shows that the results are very similar to the one-generation case with similar quantities such as r_u and r_d . Now, m_u and m_d appearing in Eqs. (11,12) are truly the masses of the first generation quarks. Once again, we find $\theta_{Weak} \propto m_\nu/m_u, m_\nu/m_d$.

The EW ν_R model [5] was first conceived to provide a testable model of the seesaw mechanism by making right-handed neutrinos non-sterile and sufficiently "light" (i.e. with a mass M_R proportional to the electroweak scale). These right-handed neutrinos do not come by themselves but are members of right-handed $SU(2)_W$ doublets which include right-handed mirror leptons. $SU(2)_W$ anomaly

freedom dictates that one should also have doublets of right-handed mirror quarks. (The model includes per family $SU(2)_W$ -singlets: e_R, u_R, d_R for the SM fermions and e_L^M, u_L^M and d_L^M for the mirror fermions.) In fact, the SM and mirror sectors allow us to evade the Nielsen-Ninomiya no-go theorem [6] which forbids the chiral SM model to be put on the lattice.

It turns out that the ingredients contained in the EW ν_R model are precisely those that allow us to solve the strong CP problem. First by being "vector-like" (SM and mirror fermions), it allows us to set $\theta_{QCD} = 0$ at tree level. Second, by mixing the left-handed SM lepton doublets with the right-handed mirror lepton doublets through the Higgs singlet fields, one obtains the neutrino Dirac mass m_D which participates in the seesaw mechanism (m_D^2/M_R). This same mixing also operates in the quark sector giving rise to mixing between SM and mirror quarks in the mass matrices, which, in turn, contributes to the CP-violating parameter $ArgDet(M_u M_d)$ in an interesting way. It vanishes if m_D goes to zero and is small ($< 10^{-10}$) because $m_D \ll M_R$ as in the seesaw mechanism. It is surprising that two seemingly unrelated phenomena find a common niche in the EW ν_R model.

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